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What is wrong with the chain ladder technique?

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Data

Table 1. Historical loss development study (1991) Automatic Facultative General Liability data (excluding asbestos and environmental).

Claim Payments

										Reserves
5012	3257	2638	898	1734	2642	1828	599	54	172	0
106	4179	1111	5270	3116	1817	-103	673	535		154
3410	5582	4881	2268	2594	3479	649	603			617
5655	5900	4211	5500	2159	2658	984				1,636
1092	8473	6271	6333	3786	225					2,747
1513	4932	5257	1233	2917						3,649
557	3463	6926	1368							5,435
1351	5596	6165								10,907
3133	2262									10,650
2063										16,339
									Overall	52,135

Development Factors

2.999	1.624	1.271	1.172	1.113	1.042	1.033	1.017	1.009

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Notation

We denote cumulative claims for accident year *i* and development year *j* by D_{ij}.
The incremental claims are denoted by

$$\left\{C_{ij} : i = 1, 2, \ldots, n; j = 0, 2, \ldots, n-i\right\}$$

The standard chain-ladder development factors are

$$f_j = \frac{\sum_{i=1}^{n-j} D_{ij}}{\sum_{i=1}^{n-j} D_{i,j-1}}$$

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Why Stochastic Reserving?

- Provides measures of variability as well as location and can provide a predictive distribution
- Enables you to move on from basic methods to more sophisticated models to tackle specific issues
- Useful in DFA analysis, ICAS etc

Common methods are Bootstrapping, Mack, Stochastic link ratios
Implicit assumption is that these are based on aggregated data, and chain-ladder type methods

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Over-dispersed Poisson

$$C_{ij} \sim Independent\ ODPoi(\mu_{ij})$$

$$\log \mu_{ij} = \eta_{ij} \qquad \mathrm{E}\left[C_{ij}\right] = \mu_{ij}$$

$$Var\left[C_{ij}\right] = \phi \mathrm{E}\left[C_{ij}\right]$$

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Predictor Structures

Some possible predictor structures are:

Chain-ladder:

$$\eta_{ij} = \mu + \alpha_i + \beta_j$$

Hoerl curve:

$$\eta_i \quad (t) = c + a_i + b.t + d \log(t)$$

Non-parametric smoothing:

$$\eta_i \quad (t) = c + a_i + s_1(t) + s_2(\log(t))$$

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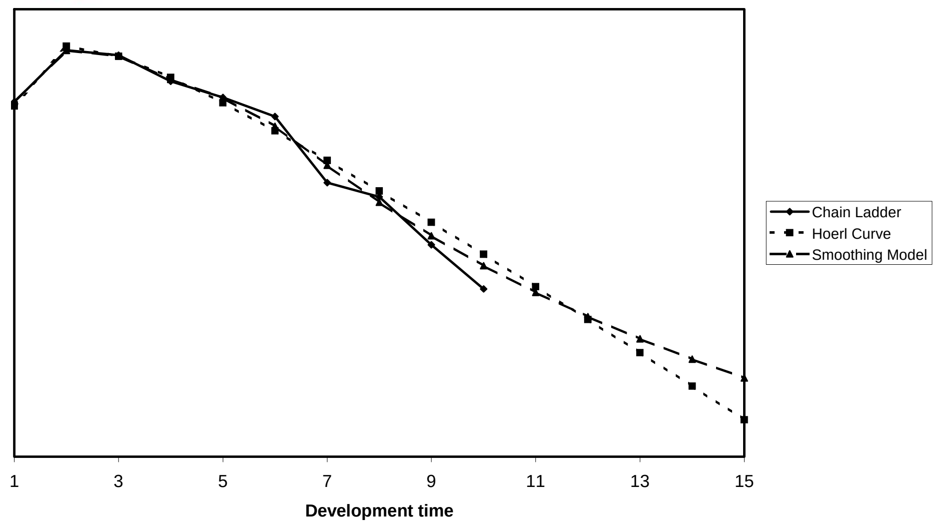

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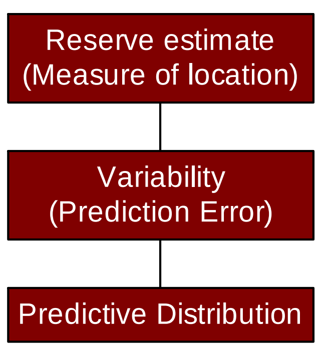
Conceptual Framework



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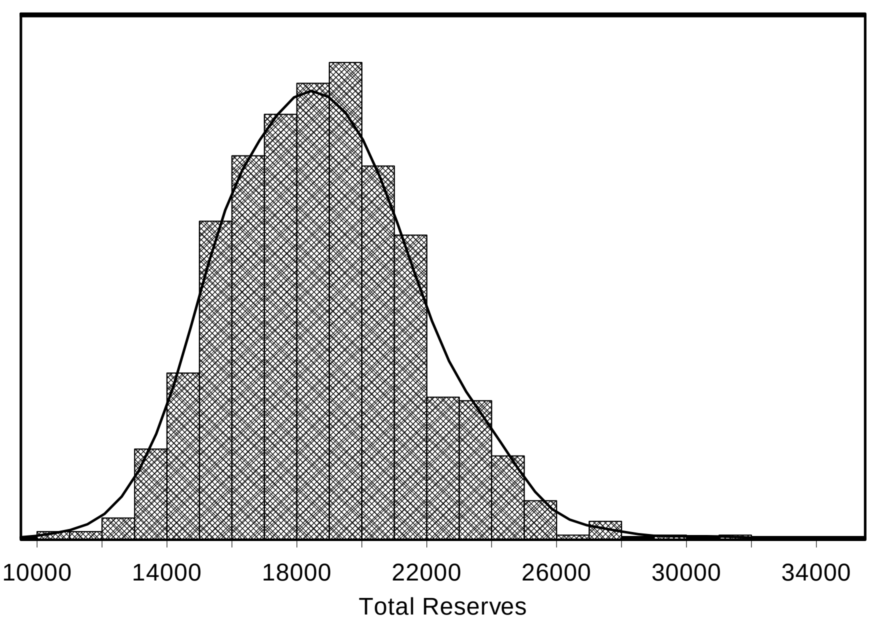
Prediction Errors

Year	Mack's Distribution Free	Over-dispersed Poisson	Bootstrap	Negative Binomial	Gamma	Log-Normal
2	80	116	117	116	48	54
3	26	46	46	46	36	39
4	19	37	36	36	29	32
5	27	31	31	30	26	28
6	29	26	26	26	24	26
7	26	23	23	22	24	26
8	22	20	20	19	26	28
9	23	24	24	23	29	31
10	29	43	43	41	37	41
Total	13	16	16	15	15	16

Figure 1. Predictive Aggregate Distribution of Total Reserves



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What is wrong

- It is simply a technique – not a model.
- If the results don't look right, you fiddle with them, rather than looking at the properties of the model.
- This becomes more acute when you consider stochastic models, because you have to fiddle with the residuals as well.
- It doesn't deal well with inflation (calendar year effects).
- It doesn't use much data.
- It doesn't (explicitly) consider the way the data are generated.

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What is wrong?

- It is a simple technique, and is easy to implement.
- If the results don't look right, it is easy to justify fiddling around with them using "actuarial judgment".
- You can also fiddle with the residuals.
- It doesn't need much data.
- You don't have to construct any complicated models to take into account the way the data are generated.
- The results usually work fairly well.

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Contents and Aims

This talk will consider some of these issues and discuss some recent developments.

There has been a lot of attention given to “stochastic models for the chain-ladder technique”. There is also quite a lot of papers on the fundamental processes driving the claims. This means that it is possible to think a bit more deeply about the models that are applied – even if compromises have to be made if the data are not completely satisfactory.

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How does the chain-ladder technique deal with inflation?

Some recent papers have considered the parameterisation of an age-period-cohort model and its application to claims reserving:

Kuang, D., B. Nielsen and J.P. Nielsen (2008a) *Identification of the age-period-cohort model and the extended chain-ladder model*. Biometrika 95, 979–986.

Kuang, D., B. Nielsen and J.P. Nielsen (2008b) *Forecasting with the age-period-cohort model and the extended chain-ladder model*. Biometrika 95, 987–991.

Brydon, D. and R. J. Verrall (2010) *Calendar Year Effects, Claims Inflation and the Chain-Ladder Technique.* Annals of Actuarial Science, to appear.

3-way model

It is assumed that the incremental claims are independent over-dispersed Poisson random variables with

$$E\left[C_{ij}\right] = \exp\left(\mu + \alpha_i + \beta_j + \gamma_{i+j-1}\right)$$

where

$$\alpha_1 = \beta_1 = \gamma_1 = \gamma_2 = 0$$

It is often assumed that chain-ladder somehow projects forward calendar year effects (inflation?) from past data into the the future.

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Experiment 1: Constant calendar year trend in the past

The first experiment considers the situation which should be best suited to the chain-ladder technique. This occurs when it is assumed that the rate of claims inflation in the data is constant.

Note that if the calendar year effect consists purely of claims inflation, then the rate of claims inflation can be measured by $\gamma_k - \gamma_{k-1}$. We set this to be a constant, and chose simple, constant values for the other parameters.

The chain-ladder technique was applied and the forecasts were analysed using the 3 –way model. The projected rate of inflation was, indeed constant and equal to the value in the past data.

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Experiment 2: Past calendar year trend not constant

"Inflation" increasing at a constant rate:

$d(\gamma_2)$	0.182
$d(\gamma_3)$	0.278
$d(\gamma_4)$	0.373
$d(\gamma_5)$	0.468
$d(\gamma_6)$	0.564
$d(\gamma_7)$	0.659
$d(\gamma_8)$	0.754
$d(\gamma_9)$	0.849
$d(\gamma_{10})$	0.945

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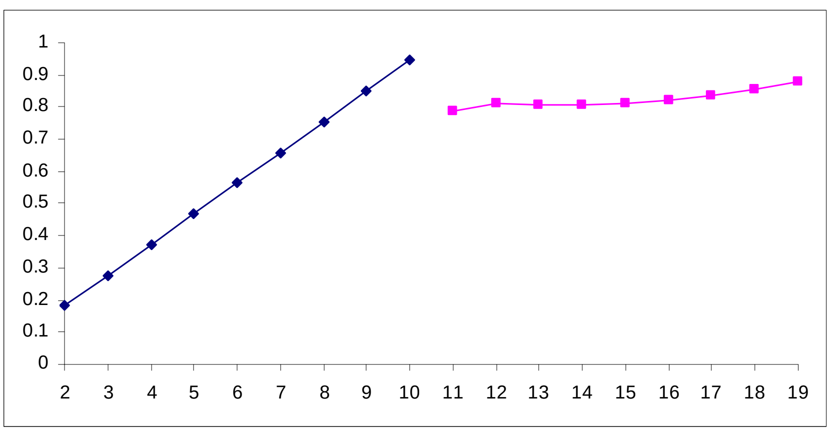
Experiment 2



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Experiment 3: A “blip”

This is similar to the first experiment, in that the rate of claims inflation in the data is constant, except for 1 point.

$d(\gamma_k) = 0.182$ for all k, except $k = m$, and $d(\gamma_m) = 0.693$

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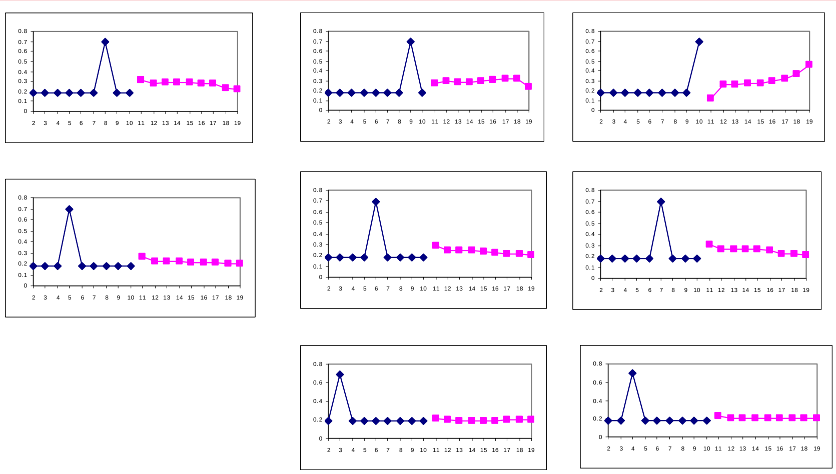
Experiment 3



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Underlying Risk Theory

Consider incremental amounts, which are made up of individual payments

$$C_{ij} = \sum_{k=1}^{N_{ij}} X_{ij}^{(k)}$$

N_{ij} is the number of individual payments

$X_{ij}^{(k)}$ is the size of an individual payments

$$E\left[C_{ij}\right] = E\left[N_{ij}\right]E\left[X_{ij}\right]$$

$$Var\left[C_{ij}\right] = E\left[N_{ij}\right]Var\left[X_{ij}\right] + Var\left[N_{ij}\right]\left(E\left[X_{ij}\right]\right)^2$$

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Standard Assumptions

Standard Assumptions for numbers and amounts are

N_{ij} have a Poisson distribution (or ODP)

X_{ij} have a distribution like a gamma distribution

This means that the variances have the following forms:

$$Var\left[N_{ij}\right] = \psi E\left[N_{ij}\right]$$

$$Var\left[X_{ij}\right] = \upsilon\left(E\left[X_{ij}\right]\right)^2$$

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$$Var\left[C_{ij}\right] = E\left[N_{ij}\right]Var\left[X_{ij}\right] + Var\left[N_{ij}\right]\left(E\left[X_{ij}\right]\right)^2$$

$$= E\left[N_{ij}\right]\upsilon\left(E\left[X_{ij}\right]\right)^2 + \psi E\left[N_{ij}\right]\left(E\left[X_{ij}\right]\right)^2$$

$$= \left[\left(\upsilon + \psi\right)E\left[X_{ij}\right]\right]E\left[N_{ij}\right]E\left[X_{ij}\right]$$

$$= \phi_{ij}E\left[C_{ij}\right]$$

$$Var\left[C_{ij}\right] = \phi_{ij}E\left[C_{ij}\right]$$

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Implications of $Var\left[C_{ij}\right] = \phi_{ij}E\left[C_{ij}\right]$

Standard ODP model had $Var\left[C_{ij}\right] = \phi E\left[C_{ij}\right]$

This may not be valid. What can we do about it? Model ϕ_{ij}

It may be reasonable to assume that ϕ_{ij} does not depend on *i*. In this case, we can replace the variance assumption in the ODP model by

$$Var\left[C_{ij}\right] = \phi_j E\left[C_{ij}\right]$$

Model the ϕ_j's

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Implications of $Var\left[C_{ij}\right] = \phi_{ij} E\left[C_{ij}\right]$

Can this modelling approach be taken any further?

Model payment numbers and amounts separately (if the data are available).

Even if the data are not available, it may be possible to build a model using this approach, and then see what it implies for aggregated data. This is something that has been looked at in the past, and it is likely to become more common as people get used to a modelling approach.

An example is the Hoerl curve.

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Claims inflation: payment year effects

Underlying many of the models and discussions so far is an implicit assumption that claims inflation has either been dealt with somehow before the data are looked at, or that the models can be applied without doing this and everything will take care of itself.

This needs to be looked at more carefully. The “separation technique” has been suggested, and recent work has examined the so-called “age-period-cohort” problem in some detail.

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Modelling the claims process

Model payment numbers and amounts separately (if the data are available).
Should we use paid or incurred data?
Verrall, Nielsen and Jessen (to appear in ASTIN) assume we have reported numbers of claims and paid aggregate claims. We build a model using the methods set out in the literature by (for example):

Bühlmann, H., Schnieper, R. and Straub, E. (1980): Claims reserves in casualty insurance based on a probability model. Mitteilungen der Vereinigung Schweizerischer Versicherungsmathematiker.
Norberg, R., (1986): A contribution to modelling of IBNR claims. Scandinavian Actuarial Journal, 155-203.
Norberg, R., (1993): Prediction of outstanding liabilities in non-life insurance. Astin Bulletin, vol. 23, no. 1, 95-115.
Norberg, R., (1999): Prediction of outstanding claims: Model variations and extensions. Astin Bulletin, vol. 29, no. 1, 5-25.

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Modelling the claims process

We have reported numbers of claims N_{ij}. We build a payment delay model for the (latent) paid numbers of claims, by first considering

$N_{ijk}^{paid} \mid N_{ij}$ which we assume has a multinomial distribution. This is the payment delay.

The number of paid claims can be found by summing these:

$$N_{ij}^{paid} = N_{ij0}^{paid} + N_{i,j-1,1}^{paid} + \ldots + N_{i,0,j}^{paid}$$

The IBNR delay is considered by modelling the incurred number of claims.
The RBNS delay is considered through the model for $N_{ijk}^{paid} \mid N_{ij}$.

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Paid claims

Denoting paid claims by X_{ij},

$$X_{ij} = \sum_{k=1}^{N_{ij}^{paid}} Y_{ij}^{(k)}$$

$$E\left[X_{ij}\right] = E\left[N_{ij}^{paid} \mid \mathbb{N}^{I}\right] E\left[Y_{ij}^{(k)}\right]$$

$$V\left[X_{ij}\right] = E\left[N_{ij}^{paid} \mid \mathbb{N}^{I}\right] V\left[Y_{ij}^{(k)}\right] + V\left[N_{ij}^{paid} \mid \mathbb{N}^{I}\right]\left(E\left[Y_{ij}^{(k)}\right]\right)^{2}$$

Assuming claims are iid

$$E\left[X_{ij}\right] = E\left[N_{ij}^{paid} \mid \mathbb{N}^{I}\right]\mu$$

$$V\left[X_{ij}\right] = E\left[N_{ij}^{paid} \mid \mathbb{N}^{I}\right]\sigma^{2} + V\left[N_{ij}^{paid} \mid \mathbb{N}^{I}\right]\mu^{2}$$

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Paid claims

$$E\left[N_{ij}^{paid} \mid \mathbb{N}^{I}\right] = E\left[\sum_{k=0}^{\min\{j,d\}} N_{i,j-k,k}^{paid} \mid \mathbb{N}^{I}\right] = \sum_{k=0}^{\min\{j,d\}} E\left[N_{i,j-k,k}^{paid} \mid \mathbb{N}^{I}\right] = \sum_{k=0}^{\min\{j,d\}} N_{i,j-k} p_k$$

$$V\left[N_{ij}^{paid} \mid \mathbb{N}^{I}\right] = \sum_{k=0}^{\min\{j,d\}} N_{i,j-k} p_k \left(1 - p_k\right)$$

$$E\left[X_{ij}\right] = \sum_{k=0}^{\min\{j,d\}} N_{i,j-k} p_k \mu$$

$$V\left[X_{ij}\right] = \sum_{k=0}^{\min\{j,d\}} N_{i,j-k} p_k \sigma^{2} + \sum_{k=0}^{\min\{j,d\}} N_{i,j-k} p_k \left(1 - p_k\right)\mu^{2}$$

$$= \sum_{k=0}^{\min\{j,d\}} N_{i,j-k} p_k \left(\sigma^{2} + \left(1 - p_k\right)\mu^{2}\right) \approx \sum_{k=0}^{\min\{j,d\}} N_{i,j-k} p_k \left(\sigma^{2} + \mu^{2}\right)$$

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Model for Paid claims

Hence, with this approximation, we can use an ODP model for paid claims: similar to chain-ladder, except there is more going on in the parameters.

$$V\left[X_{ij}\right] \approx \sum_{k=0}^{\min\{j,d\}} N_{i,j-k} p_k \left(\sigma^2 + \mu^2\right) = \varphi E\left[X_{ij}\right]$$

$$\varphi = \frac{\sigma^2 + \mu^2}{\mu^2}$$

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Applying the model

Apply a chain-ladder (Poisson or ODP) to the reported numbers of claims. The projected numbers of reported claims will allow us to analyse IBNR claims (separately).
Apply an ODP model to the triangle of paid aggregate claims. The mean is

$$E\left[X_{ij}\right] = \sum_{k=0}^{\min\{j,d\}} N_{i,j-k} \psi_k$$

$$p_k = \frac{\psi_k}{\sum_{k=0}^{d} \psi_k} \qquad \mu = \sum_{k=0}^{d} \psi_k \qquad \sigma^2 = \varphi \sum_{k=0}^{d} \psi_k - \left(\sum_{k=0}^{d} \psi_k\right)^2$$

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Data: Paid claims

i\j	0	1	2	3	4	5	6	7	8	9
1	451288	339519	333371	144988	93243	45511	25217	20406	31482	1729
2	448627	512882	168467	130674	56044	33397	56071	26522	14346	
3	693574	497737	202272	120753	125046	37154	27608	17864		
4	652043	546406	244474	200896	106802	106753	63688			
5	566082	503970	217838	145181	165519	91313				
6	606606	562543	227374	153551	132743					
7	536976	472525	154205	150564						
8	554833	590880	300964							
9	537238	701111								
10	684944									

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Data: Incurred counts

i\j	0	1	2	3	4	5	6	7	8	9
1	6238	831	49	7	1	1	2	1	2	3
2	7773	1381	23	4	1	3	1	1	3	
3	10306	1093	17	5	2	0	2	2		
4	9639	995	17	6	1	5	4			
5	9511	1386	39	4	6	5				
6	10023	1342	31	16	9					
7	9834	1424	59	24						
8	10899	1503	84							
9	11954	1704								
10	10989									

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Delay functions

j	Development Factor	IBNR Delay
0		0.8752
1	1.1353	0.1184
2	1.0038	0.0038
3	1.0009	0.0009
4	1.0003	0.0003
5	1.0003	0.0003
6	1.0002	0.0002
7	1.0001	0.0001
8	1.0003	0.0003
9	1.0004	0.0004

The average IBNR delay is 0.14 years.
The average RBNS delay is 1.52 years.

j	0	1	2	3	4	5	6	7
p	0.3637	0.2881	0.1134	0.0852	0.0661	0.0358	0.0255	0.0222

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Estimated outstanding claims

i	IBNR	RBNS	TOTAL	CHAIN LADDER
2	628	605	1,233	1,685
3	1,350	4,514	5,863	29,379
4	1,510	43,623	45,133	60,638
5	1,967	94,526	96,493	101,158
6	2,579	171,633	174,212	173,802
7	3,168	299,136	302,304	249,349
8	5,349	509,334	514,684	475,992
9	14,280	852,144	866,423	763,919
10	254,499	1,135,678	1,390,177	1,459,860
Total	285,329	3,111,192	3,396,521	3,315,779

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Conclusions

The results from an ad hoc method such as the chain-ladder technique are usually OK, but:

They are usually applied to aggregate claims without considering numbers.

Using a little more information, you can get more information out.

When considering scenarios for capital modelling, it is better to be able to look at quantities that have real meaning.

If you are not satisfied with the results from the ad hoc method, you are forced into ad hoc adjustments. If you use a more structured model, you can look at the components and make more reasoned adjustments.

Calendar year effects need to be carefully considered.